Name : ....

Fourth Semester B.Tech. Degree Examination, July 2015
(2008 Scheme)
08.401 : ENGINEERING MATHEMATICS – III (CMPUNERFHB)

Time : 3 Hours   Max. Marks : 100

Answer **all** questions from Part **A** and **one full** question from **each** module of Part **B**.

PART – A

1. Show that coshz is differentiable every where and find its derivative.

2. Show that if u and v are conjugate harmonic functions, then uv is harmonic.

3. If f(z) and $\overline{f(z)}$ are analytic, then show that f(z) is a constant.

4. Find the image of the half plane y > c under $w = \frac{1}{z}$.

5. Evaluate by Cauchy's integral formula $\int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z-1)(z-2)} dz$ where C is $|z| = 3$.

6. Expand ze^z about z = 1 as a Taylor's series.

7. Find the poles and residues of $f(z) = \frac{ze^{2iz}}{z^2+9}$.

8. Perform five iterations of the bisection method to obtain the smallest positive root of $x^3 - 5x + 1 = 0$.

9. Find the double root of the equation $x^3 - x^2 - x + 1 = 0$ by Newton Raphson method with initial approximation x = 0.8.

10. Using Lagrange's formula, find the value of y when x = 6 from the following data

x :	3	7	9	10
y :	168	120	72	63

(10×4=40 Marks)

P.T.O.

PART – B

Module – 1

11. a) Show that the function $f(z) = \frac{x^2y^3(x+iy)}{x^6+y^{10}}$, $z \neq 0$ and $f(0) = 0$, is not differentiable at $z = 0$, even though it satisfies C.R. equations.

b) If $f(z) = u + iv$ is an analytic function, prove that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$.

c) Find the bilinear transformation that maps the points $(0, 1, \infty)$ into $(-3, -1, 1)$. Find also the fixed points of the transformation.

12. a) If $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$, find $f(z) = u + iv$, which is analytic.

b) If $f(z) = u + iv$ is analytic, then u = constant and v = constant are families of curves cutting orthogonally.

c) Find the image of the circle $|z| = 2$ under $w = z + \frac{1}{z}$.

Module – 2

13. a) Evaluate $\int_{(0,0)}^{(1,1)} (z^2 + z)dz$ along two different paths and show that they are equal.

b) Find the Laurent's series expansion of $f(z) = \frac{1}{(z+2)(z^2+1)}$ in $1 < |z| < 2$.

c) Evaluate $\int_{|z-i|=2} \frac{e^z}{(z^2+4)^2} dz$.

14. a) Evaluate $\int_0^{\pi} \frac{d\theta}{a + b\cos\theta}$ where $a > |b|$.

b) Evaluate $\int_0^{\infty} \frac{1}{x^4 + a^4} dx$.

Module – 3

15. a) Using Gauss-Seidal iteration method solve the system of equations.

$$10x - 2y - z - w = 3$$
$$-2x + 10y - z - w = 15$$
$$-x - y + 10z - 2w = 27$$
$$-x - y - 2z + 10w = -9$$

b) From the data given below, find the number of students whose weight is between 60 and 70 by Newton's formula.

Weight in lbs	:	0 – 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of Students	:	250	120	100	70	50

c) The population of a town is as follows :

Year (x)	:	1941	1951	1961	1971	1981	1991
Population in lakhs (y)	:	20	24	29	36	46	51

Find the population for the year 1976.

16. a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by :

i) Trapezoidal rule

ii) Simpson's rule with 6 equal parts.

b) Using Euler's method solve numerically the equation $y' = x + y$, $y(0) = 1$. Find $y(1)$ with $h = 0.2$.

c) Compute $y(0.2)$ given $\frac{dy}{dx} + y + xy^2 = 0$, $y(0) = 1$ by taking $h = 0.2$ using Runge-Kutta method of fourth order. **(3×20=60 Marks)**